New features of the gluon and ghost propagator in the infrared region from the Gribov-Zwanziger approach

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So far, the infrared behavior of the gluon and ghost propagator based on the Gribov-Zwanziger approach predicted a positivity violating gluon propagator vanishing at zero momentum, and an infrared enhanced ghost propagator. However, recent data based on huge lattices have revealed a positivity violating gluon propagator which turns out to attain a finite *nonvanishing* value very close to zero momentum. At the same time the ghost propagator does not seem to be infrared enhanced anymore. We point out that these new features can be accounted for by yet unexploited dynamical effects within the Gribov-Zwanziger approach, leading to an infrared behavior in qualitatively good agreement with the new data.

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I. INTRODUCTION.

During the past few years, much attention has been devoted to the study of the gluon and ghost propagator, including their low energy behavior where Yang-Mills gauge theories are confining. As a consequence the gluon cannot be considered as a free particle anymore. Due to the lack of an explicit knowledge of the physical degrees of freedom, it still remains highly useful to study the gluon and ghost propagator in order to probe the nonperturbative infrared regime. Let us only mention that the gluon propagator for example finds an important use in phenomenological studies, see e.g. [1]. In the past, a good agreement between the lattice data and the analytical results arising from the Gribov-Zwanziger action in the Landau gauge were found: (1) an infrared suppressed and positivity violating gluon propagator vanishing at zero momentum, (2) an infrared enhanced ghost propagator. We recall that the Gribov-Zwanziger action was constructed to take into account the existence of gauge copies [2, 3]. However, very recent lattice data obtained at large volumes [4, 5], which allows one to get very close to zero momentum, now give evidence of a hitherto unexpected behavior in the deep infrared: (1) an infrared suppressed and positivity violating gluon propagator *nonvanishing at zero momentum*, (2) a ghost propagator essentially behaving like $1/p^2$ at low momentum, which is clearly *not enhanced*. To our knowledge, none of the current theoretical approaches exhibit all such features [6, 7, 8, 9, 10]. In this letter we propose a dynamical mechanism within the Gribov-Zwanziger approach that could account for the new lattice results.

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II. THE GRIBOV-ZWANZIGER ACTION.

We first give a short overview of the action constructed by Zwanziger [3] which implements the restriction to the Gribov region Ω in Euclidean Yang-Mills theories in the Landau gauge. We recall that this restriction to Ω can be implemented by adding the nonlocal horizon function to the original Yang-Mills action,

$$S_{\rm YM}+S_{\rm Landau}-\gamma^4 g^2\int {\rm d}^4x f^{abc}A^b_\mu\left(\mathcal{M}^{-1}\right)^{ad} f^{dec}A^e_\mu\,, \qquad (1)$$

where $\mathcal{M}^{ab}=-\partial_\mu\left(\partial_\mu\delta^{ab}+gf^{acb}A^c_\mu\right)$ is the Faddeev-Popov operator, $S_{\rm YM}=1/4\int {\rm d}^4x F_{\mu\nu}F_{\mu\nu}$ and $S_{\rm Landau}=\int {\rm d}^4x(b^a\partial_\mu A^a_\mu+\overline{c}^a\partial_\mu D^{ab}_\mu c^b)$ stands for the gauge fixing and the ghost part. However, it is unclear how to handle consistently such a nonlocal action at the quantum level, so we are obliged to add extra fields $\left(\overline{\varphi}^{ac}_\mu,\varphi^{ac}_\mu,\overline{\omega}^{ac}_\mu,\omega^{ac}_\mu\right)$ in order to localize this action. Doing so, the Gribov-Zwanziger action reads [3, 10]

$$S=S_0-\gamma^2 g\int {\rm d}^4x\left(f^{abc}A^a_\mu\varphi^{bc}_\mu+f^{abc}A^a_\mu\overline{\varphi}^{bc}_\mu+\frac{4}{g}\left(N^2-1\right)\gamma^2\right)\,, \qquad (2)$$

with

$$\begin{aligned} S_0 &= S_{\rm YM}+\int {\rm d}^4x\,\left(b^a\partial_\mu A^a_\mu+\overline{c}^a\partial_\mu\left(D_\mu c\right)^a\right)\\ &+\int {\rm d}^4x\Big(\overline{\varphi}^a_i\partial_\nu\left(D_\nu\varphi_i\right)^a-\overline{\omega}^a_i\partial_\nu\left(D_\nu\omega_i\right)^a\\ &\quad -g\left(\partial_\nu\overline{\omega}^a_i\right)f^{abm}\left(D_\nu c\right)^b\varphi^m_i\Big)\,, \end{aligned} \qquad (3)$$

whereby $\left(\overline{\varphi}^{ac}_\mu,\varphi^{ac}_\mu\right)$ are a pair of complex conjugate bosonic fields, whereas $\left(\overline{\omega}^{ac}_\mu,\omega^{ac}_\mu\right)$ are anticommuting. Due to a global $U(f)$ symmetry, $f=4\left(N^2-1\right)$, with respect to the composite index $i=(\mu,c)$ of the additional fields $\left(\overline{\varphi}^{ac}_\mu,\varphi^{ac}_\mu,\overline{\omega}^{ac}_\mu,\omega^{ac}_\mu\right)$, we introduced a notational shorthand,

$$\left(\overline{\varphi}^{ac}_\mu,\varphi^{ac}_\mu,\overline{\omega}^{ac}_\mu,\omega^{ac}_\mu\right)=\left(\overline{\varphi}^a_i,\varphi^a_i,\overline{\omega}^a_i,\omega^a_i\right)\,. \qquad (4)$$

The dimensional parameter γ is not free, being determined by the gap equation (horizon condition) $\partial\Gamma/\partial\gamma = 0$, which ensures the restriction to the Gribov region. Γ is the quantum effective action obtained from (2).
As it has been shown in [3, 10], the action (2) is renormalizable to all orders. To prove this by the method of algebraic renormalization [11], we embed this action into a larger one which contains more symmetries. This action turns out to be given by [10]

$$\Sigma = S_0 + S_{\rm s} + S_{\rm ext} \,, \tag{5}$$

with S_0 given in (3) and

$$\begin{aligned} S_{\rm s} &= s\int d^4x \left(-U^{ai}_\mu (D_\mu\varphi_i)^a - V^{ai}_\mu (D_\mu\overline{\omega}_i)^a - U^{ai}_\mu V^{ai}_\mu\right) \,, \\ S_{\rm ext} &= \int d^4x \left(-K^a_\mu (D_\mu c)^a + \frac{1}{2} g L^a f^{abc} c^b c^c\right) \,. \end{aligned} \tag{6}$$

We introduced new sources M^{ai}_μ, V^{ai}_μ ,U^{ai}_μ, N^{ai}_μ, K^a_μ and L^a, which are necessary to analyze the renormalization of the corresponding composite field operators in a BRST invariant fashion. The BRST operator s acts on the fields and sources appearing in the action as follows

$$\begin{aligned} & sA^a_\mu = -(D_\mu c)^a \,, sc^a = \frac{1}{2} g f^{abc} c^b c^c \,, s\overline{c}^a = b^a \,, sb^a = 0 \,, \\ & s\varphi^a_i = \omega^a_i \,, s\omega^a_i = 0 \,, s\overline{\omega}^a_i = \overline{\varphi}^a_i \,, s\overline{\varphi}^a_i = 0 \,, sU^{ai}_\mu = M^{ai}_\mu \,, \\ & sM^{ai}_\mu = 0 \,, sV^{ai}_\mu = N^{ai}_\mu \,, sN^{ai}_\mu = 0 \,, sK^a_\mu = 0, sL^a = 0 \,, \end{aligned} \tag{7}$$

whereby the BRST operator s is nilpotent, $s^2 = 0$. One can easily see that the action Σ is indeed BRST invariant. Henceforth, the action Σ displays a greater number of symmetries, encoded in the following Ward identities [3, 10].

- For the $U(f)$ invariance mentioned before we have

$$\begin{aligned} U_{ij}\Sigma &= 0 \,, \\ U_{ij} &= \int d^4x \left(\varphi^a_i \frac{\delta}{\delta\varphi^a_j} - \overline{\varphi}^a_j \frac{\delta}{\delta\overline{\varphi}^a_i} + \omega^a_i \frac{\delta}{\delta\omega^a_j} - \overline{\omega}^a_j \frac{\delta}{\delta\overline{\omega}^a_i}\right) \,. \end{aligned} \tag{8}$$

- The Slavnov-Taylor identity reads

$$\mathcal{S}(\Sigma) = 0 \,, \tag{9}$$

$$\begin{aligned} \mathcal{S}(\Sigma) = \int d^4x \Bigg(& \frac{\delta\Sigma}{\delta K^a_\mu}\frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + b^a \frac{\delta\Sigma}{\delta\overline{c}^a} + \overline{\varphi}^a_i \frac{\delta\Sigma}{\delta\overline{\omega}^a_i} \\ & + \omega^a_i \frac{\delta\Sigma}{\delta\varphi^a_i} + M^{ai}_\mu \frac{\delta\Sigma}{\delta U^{ai}_\mu} + N^{ai}_\mu \frac{\delta\Sigma}{\delta V^{ai}_\mu}\Bigg) \,. \end{aligned}$$

- The Landau gauge condition and the antighost equation are given by

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A^a_\mu \,, \tag{10}$$

$$\frac{\delta\Sigma}{\delta\overline{c}^a} + \partial_\mu \frac{\delta\Sigma}{\delta K^a_\mu} = 0 \,. \tag{11}$$

- The ghost Ward identity:

$$\begin{aligned} \mathcal{G}^a\Sigma &= \Delta^a_{\rm cl} \,, \\ \mathcal{G}^a &= \int d^4x \Bigg(\frac{\delta}{\delta c^a} + g f^{abc}\Bigg(\overline{c}^b \frac{\delta}{\delta b^c} + \varphi^b_i \frac{\delta}{\delta\omega^c_i} \\ & \qquad + \overline{\omega}^b_i \frac{\delta}{\delta\overline{\varphi}^c_i} + V^{bi}_\mu \frac{\delta}{\delta N^{ci}_\mu} + U^{bi}_\mu \frac{\delta}{\delta M^{ci}_\mu}\Bigg)\Bigg) \,, \\ \Delta^a_{\rm cl} &= g\int d^4x f^{abc}\left(K^b_\mu A^c_\mu - L^b c^c\right) \,. \end{aligned} \tag{12}$$

The term $\Delta^a_{\rm cl}$ denotes a classical breaking as it is linear in the quantum fields.

- The linearly broken local constraints:

$$\frac{\delta\Sigma}{\delta\varphi^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta M^{ai}_\mu} = g f^{abc} A^b_\mu V^{ci}_\mu + J\varphi^a_i \,, \tag{13}$$

$$\frac{\delta\Sigma}{\delta\omega^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta N^{ai}_\mu} - g f^{abc}\overline{\omega}^{bi}\frac{\delta\Sigma}{\delta b^c} = g f^{abc} A^b_\mu U^{ci}_\mu + J\overline{\omega}^a_i \,, \tag{14}$$

$$\frac{\delta\Sigma}{\delta\overline{\omega}^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta U^{ai}_\mu} - g f^{abc} V^{bi}_\mu \frac{\delta\Sigma}{\delta K^c_\mu} = -g f^{abc} A^b_\mu N^{ci}_\mu - J\omega^a_i \,, \tag{15}$$

$$\begin{aligned} & \frac{\delta\Sigma}{\delta\overline{\varphi}^{ai}} + \partial_\mu \frac{\delta\Sigma}{\delta V^{ai}_\mu} - g f^{abc}\overline{\varphi}^{bi}\frac{\delta\Sigma}{\delta b^c} - g f^{abc}\overline{\omega}^{bi}\frac{\delta\Sigma}{\delta\overline{c}^c} \\ & \qquad - g f^{abc} U^{bi}_\mu \frac{\delta\Sigma}{\delta K^c_\mu} = g f^{abc} A^b_\mu M^{ci}_\mu + J\overline{\varphi}^a_i \,. \end{aligned} \tag{16}$$

- The exact $\mathcal{R}_{ij}$ symmetry:

$$\begin{aligned} \mathcal{R}_{ij}\Sigma &= 0 \,, \\ \mathcal{R}_{ij} &= \int d^4x \left(\varphi^a_i \frac{\delta}{\delta\omega^a_j} - \overline{\omega}^a_j \frac{\delta}{\delta\overline{\varphi}^a_i} - V^{ai}_\mu \frac{\delta}{\delta N^{aj}_\mu} + U^{aj}_\mu \frac{\delta}{\delta M^{ai}_\mu}\right) \,. \end{aligned} \tag{17}$$

According to the algebraic renormalization procedure [11], Ward identities induce constraints on the most general allowed counterterm Σ^c at the quantum level. Once Σ^c is found, one can check if it can be reabsorbed in the original starting action by a suitable renormalization of fields, sources and parameters, thereby establishing the renormalizability. One can show [10] that Σ^c does not depend on the Lagrange multiplier b^a, and that the antighost $\overline{c}^a$ and the i-valued fields φ^a_i, ω^a_i, $\overline{\varphi}^a_i$, $\overline{\omega}^a_i$ can enter only through the combinations

$$\begin{aligned} & \widetilde{K}^a_\mu = K^a_\mu + \partial_\mu\overline{c}^a - g f^{abc}\widetilde{U}^{bi}_\mu\varphi^{ci} - g f^{abc} V^{bi}_\mu\overline{\omega}^{ci} \,, \\ & \widetilde{U}^{ai}_\mu = U^{ai}_\mu + \partial_\mu\overline{\omega}^{ai} \,, \widetilde{V}^{ai}_\mu = V^{ai}_\mu + \partial_\mu\varphi^{ai} \,, \\ & \widetilde{N}^{ai}_\mu = N^{ai}_\mu + \partial_\mu\omega^{ai} \,, \widetilde{M}^{ai}_\mu = V^{ai}_\mu + \partial_\mu\overline{\varphi}^{ai} \,. \end{aligned} \tag{18}$$

Imposing the constraints, the most general counterterm yields,

$$\begin{aligned} \Sigma^c &= a_0 S_{YM} \\ & + a_1 \int d^4x \left(A^a_\mu \frac{\delta S_{YM}}{\delta A^a_\mu} + \widetilde{K}^a_\mu \partial_\mu c^a + \widetilde{V}^{ai}_\mu \widetilde{M}^{ai}_\mu - \widetilde{U}^{ai}_\mu \widetilde{N}^{ai}_\mu\right) \end{aligned} \tag{19}$$

with a_0, a_1 two arbitrary parameters. It then turns out that Σ^c can be reabsorbed into the starting action (5) by a multiplicative renormalization [3, 10]. At the end, we give the sources the following physical values

$$\begin{aligned} M^{ab}_{\mu\nu}\big|_{phys} = V^{ab}_{\mu\nu}\big|_{phys} = \gamma^2\delta^{ab}\delta_{\mu\nu} \,, \\ U^{ai}_{\mu}\big|_{phys} = N^{ai}_{\mu}\big|_{phys} = K^{a}_{\mu}\big|_{phys} = L^{a}\big|_{phys} = 0 \,, \end{aligned} \tag{20}$$

in order to recover the physical action (2).

III. INCLUSION OF A NEW DYNAMICAL EFFECT.

In a sense, the fields $\left(\overline{\varphi}^{ac}_{\mu}, \varphi^{ac}_{\mu}, \overline{\omega}^{ac}_{\mu}, \omega^{ac}_{\mu}\right)$ introduced to localize the horizon function appearing in (1), will correspond to the nonlocal dynamics. Once these fields are present, they will quite evidently develop their own dynamics at the quantum level, which might include further nonperturbative effects, not yet accounted for. These effects can induce important additional changes in the infrared region. More precisely, looking at the $A\varphi$-coupling present at tree level in the action (2), a nontrivial effect in the φ-sector will get immediately translated into the gluon sector. We shall now explore the effects of a dynamical mass generation for the φ-fields. This can be done by introducing the local composite operator $\overline{\varphi}\varphi$ into the action (2). Since the horizon condition is in fact equivalent with giving a particular value to a dimension 2 $A\varphi$-condensate [3], more precisely $\langle g f^{abc} A^a_\mu(\varphi^{bc}_\mu + \overline{\varphi}^{bc}_\mu)\rangle = -2d\left(N^2-1\right)\gamma^2$, it does seem to be reasonably fair to consider a possible $\overline{\varphi}\varphi$-condensation. Remarkably, it turns out that this is possible while preserving the renormalizability and BRST invariance. In order to do so, we try to enlarge the action Σ by adding a massive term of the form $J\overline{\varphi}^a_i\varphi^a_i$, with J a new source. First of all, for renormalization purposes, we have to add this term in a BRST invariant way. Secondly, in analogy with [10], we will also need a term $\propto J^2$, indispensable to kill potential novel divergences $\propto J^2$ in the generating functional. We thus consider the following extended action:

$$\Sigma' = \Sigma + S_{\overline{\varphi}\varphi}, \tag{21}$$

$$\begin{aligned} S_{\overline{\varphi}\varphi} &= \int d^4x \left[s(-J\overline{\omega}^a_i\varphi^a_i) + \rho\frac{J^2}{2}\right] \\ &= \int d^4x \left[-J\left(\overline{\varphi}^a_i\varphi^a_i - \overline{\omega}^a_i\omega^a_i\right) + \rho\frac{J^2}{2}\right] \,, \end{aligned} \tag{22}$$

with ρ a new dimensionless quantity and J a new source invariant under the BRST transformation, $sJ=0$. We underline that the final mass operator, $\overline{\varphi}\varphi - \overline{\omega}\omega$, is BRST invariant. Now, it can be nicely checked that all the Ward identities of the previous section are maintained. The final output is that the new action (21) enjoys multiplicative renormalizability [12].
An interesting feature is that the anomalous dimension of the mass J is not an independent quantity, as it is related to the running of the gauge coupling and of the gluon field [12].
We mention already that we will be able to prove that this new parameter ρ is in fact redundant. We postpone this to a forthcoming larger paper [12], since the aim of this letter is merely to illustrate the relevance of the introduced mass operator.

Summarizing, the BRST invariant mass operator $\overline{\varphi}\varphi - \overline{\omega}\omega$ fits quite naturally into the theory: it is renormalizable to all orders and, moreover, it does not introduce any new renormalization constants into the theory.

IV. THE MODIFIED GLUON AND GHOST PROPAGATOR.

Finally, we come to the main purpose of this letter. We shall have a look at the effect on the propagators in the presence of the mass operator $\overline{\varphi}\varphi - \overline{\omega}\omega$.

A. The gluon propagator.

In order to calculate the gluon propagator we only need the quadratic part of the action Σ'. We also replace the source J with the more conventional mass notation M^2, so that

$$\begin{aligned} \Sigma'_0 &= \int d^4x \Big(\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \frac{1}{2\alpha}\left(\partial_\mu A^a_\mu\right)^2 + \overline{\varphi}^{ab}_\mu \partial^2 \varphi^{ab}_\mu \\ &\quad -\gamma^2 g(f^{abc}A^a_\mu\varphi^{bc}_\mu + f^{abc}A^a_\mu\overline{\varphi}^{bc}_\mu) - M^2\overline{\varphi}^{ab}_\mu\varphi^{ab}_\mu \Big) + \ldots \,, \end{aligned}$$

where the limit $\alpha \to 0$ is understood in order to recover the Landau gauge. The "..." stand for the constant term $-d(N^2-1)\gamma^4$ and other terms in the ghost and $\omega, \overline{\omega}$ fields irrelevant for the calculation of the gluon propagator. Next, we integrate out φ and $\overline{\varphi}$, yielding

$$\begin{aligned} \Sigma'_0 &= \int d^4x \frac{1}{2} A^a_\mu \Delta^{ab}_{\mu\nu} A^b_\nu + \ldots \,, \\ \Delta^{ab}_{\mu\nu} &= \left[\left(-\partial^2 - \frac{2g^2N\gamma^4}{\partial^2 - M^2}\right)\delta_{\mu\nu} + \partial_\mu\partial_\nu\left(\frac{1}{\alpha}-1\right)\right]\delta^{ab}. \end{aligned}$$

Taking the inverse of $\Delta^{ab}_{\mu\nu}$ and converting it into momentum space, we find the following gluon propagator

$$\mathcal{D}^{ab}_{\mu\nu}(p) = \underbrace{\frac{p^2+M^2}{p^4+M^2p^2+2g^2N\gamma^4}}_{\mathcal{D}(p)} \mathcal{P}_{\mu\nu}(p)\delta^{ab} \,, \tag{23}$$

where $\mathcal{P}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$. From this expression we make three observations: (1) $\mathcal{D}(p)$ enjoys infrared suppression, (2) $\mathcal{D}(p)$ displays a positivity violation, (3) $\mathcal{D}(0) \propto M^2$, so the gluon propagator does not vanish at the origin, which is clearly a different result due to the novel mass term proportional to $\overline{\varphi}\varphi - \overline{\omega}\omega$.

B. The ghost propagator.

Let us now turn to the ghost propagator. We first derive the gap equation for the Gribov parameter γ, useful for the calculation of this propagator at one loop. The part of the one loop

FIG. 1: The one loop corrected ghost propagator.

effective action $\Gamma^{(1)}$ relevant for this gap equation reads

$$\Gamma^{(1)}_{\gamma} = -d(N^2-1)\gamma^4 + \frac{(N^2-1)}{2}(d-1)\int \frac{\mathrm{d}^d p}{(2\pi)^d} \ln\left(p^4 + p^2\frac{2Ng^2\gamma^4}{p^2+M^2}\right).$$

Minimizing $\Gamma^{(1)}_{\gamma}$ with respect to γ and setting $d=4$ leads to the following gap equation:

$$\frac{4}{3g^2N} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p^4+M^2p^2+2g^2N\gamma^4}\,. \qquad (24)$$

We are now ready to compute the ghost propagator at one loop order, as depicted in FIG.1. The corresponding analytical representation reads

$$\mathcal{G}^{ab}(k) = \delta^{ab}\frac{1}{k^2}\frac{1}{1-\sigma}\,, \qquad (25)$$

with

$$\sigma = Ng^2\frac{k_\mu k_\nu}{k^2}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{(k-q)^2}\frac{q^2+M^2}{q^4+M^2q^2+2g^2N\gamma^4}\mathcal{P}_{\mu\nu}(q)\,.$$

If we take a closer look at the integral appearing in σ, we can invoke the gap equation (24) in order to simplify $(1-\sigma)$. By virtue of

$$Ng^2\frac{k_\mu k_\nu}{k^2}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{q^4+M^2q^2+2g^2N\gamma^4}\left[\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right] = Ng^2\frac{k_\mu k_\nu}{k^2}\cdot\frac{3}{4}\delta_{\mu\nu}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{q^4+M^2q^2+2g^2N\gamma^4} = 1,$$

we find

$$1-\sigma = Ng^2\frac{k_\mu k_\nu}{k^2}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\left[\frac{k^2-2k\cdot q-M^2}{(k-q)^2}\right] \times \frac{1}{q^4+M^2q^2+2g^2N\gamma^4}\mathcal{P}_{\mu\nu}(q)\,. \qquad (26)$$

The last expression reveals that the ghost propagator will not be enhanced at $k^2=0$. Indeed, if we expand $(1-\sigma)$ in the region around $k^2=0$, we see

$$1-\sigma = Ng^2\frac{k_\mu k_\nu}{k^2}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{-M^2}{q^2}\frac{\mathcal{P}_{\mu\nu}(q)}{q^4+M^2q^2+2g^2N\gamma^4} + \mathcal{O}(k^2) = -\frac{3}{4}g^2M^2\frac{1}{8\pi^2}\frac{\ln\left(\frac{M^2+\sqrt{M^4-8g^2N\gamma^4}}{M^2-\sqrt{M^4-8g^2N\gamma^4}}\right)}{2\sqrt{M^4-8g^2N\gamma^4}} + \mathcal{O}(k^2)\,. \qquad (27)$$

We conclude that the ghost propagator keeps displaying a $1/k^2$ behavior for $k^2\approx 0$. It becomes apparent now that, without the introduction of the new BRST invariant mass term, the Gribov-Zwanziger approach would predict a $1/k^4$ instead of a $1/k^2$ behavior.

V. CONCLUSION.

In this letter, we have pointed out that the new lattice data for the gluon and ghost propagator have a simple understanding within the Gribov-Zwanziger approach. This is due to the introduction of a multiplicatively renormalizable BRST invariant mass operator $\overline{\varphi}\varphi-\overline{\omega}\omega$, which fits into the theory in a very natural way. We hope that the theoretical framework presented here will stimulate further investigations, allowing a deeper understanding of the propagators in the low momentum region. We end by noticing that it would be interesting to find out what the analogous effects in the maximal Abelian gauge might be and also compare those with available lattice data [13].

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